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A measurement of Lagrangian velocity autocorrelation in approximately isotropic turbulence

By D. J. SHLIEN† AND S. CORRSIN
Department of Mechanics and Materials Science,
The Johns Hopkins University

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By measuring the heat dispersion behind a heated wire stretched across a wind tunnel (Taylor 1921, 1935), the Lagrangian velocity autocorrelation was determined in an approximately isotropic, grid-generated turbulent flow. The techniques were similar to previous ones, but the scatter is less. Assuming self-preservation of the Lagrangian velocity statistics in a form consistent with recent measurements of decay in this flow (Comte-Bellot & Corrsin 1966, 1971), a stationary and an approximately self-preserving form for the dispersion were derived and approximately verified over the range of the experiment.

Possibly the most important aspect of this experiment is that data were available in the same flow on the simplest Eulerian velocity autocorrelation in time, the correlation at a fixed spatial point translating with the mean flow (Comte-Bellot & Corrsin 1971). Thus, the Lagrangian velocity autocorrelation coefficient function calculated from the dispersion data could be compared with this corresponding Eulerian function. It was found that the Lagrangian Taylor microscale is very much larger than the analogous Eulerian microscale (76 ms compared with 6·2 ms), contrary to an estimate of Corrsin (1963). The Lagrangian integral time scale is roughly equal to the Eulerian one, being larger by about 25 %.

1. Introduction

One of the most striking properties of turbulence is its ability to disperse fluid particles. Not only is turbulent dispersion intrinsically interesting, but it is fundamental to heat and mass transport problems. The most natural co-ordinates for dispersion studies are the material (Lagrangian) ones, but Lagrangian statistical functions are considerably less accessible, both theoretically and experimentally, than the corresponding spatial (Eulerian) ones. Thus, experimental comparisons of statistics in the two kinds of co-ordinates can prove to be enlightening and useful.

Ever since Taylor's (1921) classic work on 'diffusion by continuous movements' attempts have been made to compute the Lagrangian velocity autocorrelation function from dispersion measurements in isotropic flow. Typical were the moderately successful measurements of Uberoi & Corrsin (1953), where scatter made the calculations of Lagrangian correlations very rough indeed.

† Present address: School of Engineering, Tel-Aviv University, Israel.

Townsend (1954) did not have sufficient confidence in his dispersion measurements to extract Lagrangian correlations, but he was successful in collapsing the Uberoi/Corrsin data onto a single curve by assuming similarity.

Mickelsen's (1955) measurements of dispersion in the core region of a duct are sometimes quoted in atmospheric dispersion literature as evidence that the Lagrangian and Eulerian correlations are similar in shape. This inference would be incorrect, because the measured standard deviation of the concentration profile was linear with downstream distance for the entire range of his measurements, therefore resulting in a Lagrangian correlation coefficient approximately equal to 1·0.

Among the several other experiments in 'isotropic' flows, that of Micheli (1968) is particularly interesting. He measured the dispersion behind a heated wire in a grid-generated turbulent flow of water. One of his objectives was to confirm Saffman's (1960, 1962) analysis for the interaction of molecular diffusion with the turbulent diffusion by using a fluid (water) of molecular diffusivity different from those of previous measurements (air, helium and carbon dioxide). However, his confirmation appears to be unconvincing because of relatively large scatter. Similarly, Mickelsen's (1960) data exhibited scatter of the same order of magnitude as the molecular diffusion estimated by assuming statistical independence of molecular and turbulent phenomena. Micheli also calculated the Lagrangian velocity autocorrelation coefficient. There appears to be a discrepancy between his measured dispersion and the calculated Lagrangian microscale, his computed microscale appearing to be too small by a factor of 2 or 3.

An earlier empirical estimate of the effect of molecular diffusion was made by Kistler (1956). By measuring the temperature cross-correlation function perpendicular to the mean flow and perpendicular to the tagging wire, he obtained a measure of the mean thickness of the 'flapping' hot-air sheet.

Townsend's (1951) original detailed discussion on the initial effect of molecular diffusion on a small contaminant (e.g. heat) spot was corrected by Saffman (1960, 1962). Tennekes & Lumley (1972, p. 242) also considered the problem, although they said nothing about Saffman's interesting result that for very small times the turbulent–molecular interaction can actually reduce the gross molecular diffusion below the amount for molecular conduction alone.

In one of the most recent measurements of dispersion of foreign particles in grid-generated turbulence, Snyder & Lumley (1971) estimated that their smallest and lightest particles moved essentially like fluid particles, thus allowing calculation of the Lagrangian velocity correlation from the particle dispersion. The data appear to be too scattered at small times to allow them to estimate the Lagrangian microscale.

In addition to measurements of mean temperature, the r.m.s. values, skewness, micro- and integral scales and probability distribution of temperature fluctuations were measured in a turbulent boundary layer as well as in the grid-generated turbulence. The probability distribution of pulse and gap widths of the temperature signal was also measured (Shlien 1971), and will be published in the future.

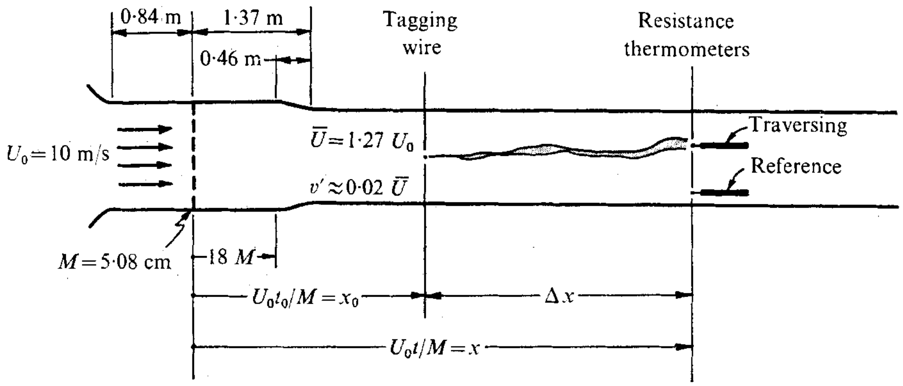

FIGURE 1. Co-ordinate system.

2. Fluid mechanical apparatus

The closed-circuit wind tunnel used in this experiment and the flow in its test section are described by Comte-Bellot & Corrsin (1966, 1971). Background 'turbulence' levels in the test section, $1{\cdot}0 \times 1{\cdot}3$ m in cross-section by 10 m long, are less than 0·1 %. To equilibrate the average turbulent kinetic energies of the grid-generated streamwise and cross-stream components, the tunnel was designed with a 1·27 : 1·0 secondary contraction downstream of the grid (figure 1).

Since most of the Eulerian data reported by Comte-Bellot & Corrsin (1971) were taken in the turbulence generated by a biplane, square rod, polished dural grid with a 5·08 cm mesh M, and solidity 0·34, the same grid and conditions were used in these measurements. The mean velocity U_0 approaching the grid was 10 m/s and thus the test section mean velocity $\bar{U}$ was 12·7 m/s. The Reynolds number R_λ, based on the Taylor transverse microscale and r.m.s. component velocity, decreased from 72 at the beginning of the test section to 61 near the end.

Comte-Bellot & Corrsin also reported various velocity correlations in space and time, including the most basic Eulerian velocity double correlation in time, at a point moving with the mean flow. This is essentially an Eulerian autocorrelation. Thus it will be possible to compare this with the measured Lagrangian velocity autocorrelation in time.

3. Measuring equipment and procedures

A platinum wire (diameters of 0·0127 and 0·127 mm were used), stretched across the wind tunnel, was heated electrically with direct current to almost glowing, thereby tagging fluid particles. Overheat ratios of 0·3–0·5 were used and tension was maintained in the wire by suspending a weight from it. Using the 'film' temperature (average of wire and ambient temperatures) to identify an effective kinematic viscosity, the maximum Reynolds number was computed to be less than 40 in all cases, i.e. below the critical (vortex shedding) Reynolds number. The concept of an apparent source location was used to compensate for finite tagging-wire diameter. As a check on the effect of tagging-wire diameter, some measurements were repeated with the two sizes of wire. The smaller diameter tagging wire was normally used for the 'short-time' dispersion measurements.

Temperature was measured using 90 % platinum/10 % rhodium resistance thermometers having a sensing element of diameter 0·63 μm, and two lengths – 5 mm (resistance 3 kΩ) and 0·5 mm. A slight bow was deliberately put in the wire to improve its lifetime considerably over those of straight wires (having some tension). A Shapiro & Edwards current control panel was used as the current source ($\sim$ 0·2 mA) and as a means of subtracting the signals from two resistance thermometers – one traversing, the other sensing ambient temperature in the test section, outside the thermal wake region.

The output from the current control panel was passed through a Honeywell, model A 20 B, d.c. amplifier and then integrated for 90 s (set on a Cramer clock) using a calibrated chemical integrator (Model SI 100, Self-Organizing Systems Inc.) to obtain the mean. Typical drifts resulted in an apparent temperature change of 0·003 °C. Where accuracy demanded it, the temperature difference was measured with the tagging wire cold, then hot, and immediately afterwards cold again. The average of the first and last readings was subtracted from the one with the tagging wire hot, to reduce the error due to drift.

4. Relationship between dispersion and Lagrangian correlation

In a decaying isotropic turbulent field, the Lagrangian correlation $R(t, t_0)$ has been shown to be related to the dispersion $\overline{Y^2}(t, t_0)$ by Townsend (1954):

$$R(t, t_0) = -\partial^2[\tfrac{1}{2}\overline{Y^2}(t, t_0)]/\partial t\, \partial t_0,$$

where t_0 is the tagging time. (Because of the use of a secondary contraction in the wind tunnel, all times are to be considered travel times from the grid

$$t = \int_0^x \frac{dx}{\overline{U}(x)}$$

unless indicated otherwise.) To reduce the number of necessary measurements, Townsend (1954) assumed self-preservation of particle (i.e. Lagrangian) velocity statistics. With this assumption and effectively the Batchelor/Townsend (1948) energy decay and scale growth laws,

$$v'^2(t) \propto (\Delta t)^{-1}, \quad L_g(t) \propto (\Delta t)^{\frac{1}{2}},$$

he found a local time scale $t_s(t) = L_g(t)/v'(t) \propto \Delta t$, where $v'(t)$ is the r.m.s. turbulent (Eulerian) velocity component in the y direction. Using this assumption for time scaling, he deduced the following expression for the Lagrangian velocity correlation:

$$R(t, t_0) = \frac{\Delta t}{\Delta t_0} \frac{\partial^2}{\partial t^2} [\tfrac{1}{2}\overline{Y^2}(t, t_0)].$$

The Δ indicates that the time of the apparent origin of the turbulence is subtracted, i.e. $\Delta t \equiv t - t^*$, with t^* the (extrapolated) value of t at which $v'^{-1} = 0$. Thus with the self-preservation assumption, the Lagrangian correlation may be calculated by measuring the dispersion function for only a single tagging time. Several studies have since used Townsend's 'renormalization', notably that of Micheli (1968) and Snyder & Lumley (1971; they reference some unpublished applications by other investigators).

However, it has been reasonably well established both experimentally (Comte-Bellot & Corrsin 1966) and theoretically (Saffman 1967) that better power-law approximations for decay are more like

$$v'^2(t) \propto (\Delta t)^{-1\cdot 25}, \quad L_g(t) \propto (\Delta t)^{0\cdot 4},$$

where L_g is the transverse Eulerian integral length scale, so that the local time scale $= L_g(t)/v'(t) \propto (\Delta t)^{1\cdot 025} \approx \Delta t$. Thus the time rescaling here is identical to that of Townsend, although justified by different decay behaviour. Using these measured decay laws together with the assumption of self-preservation of particle velocity statistics, it can be shown (Shlien 1971) that

$$\mathscr{R}_L(\tau) = d^2[\tfrac{1}{2}Z^2(\tau)]/d\tau^2, \tag{1}$$

where

$$\mathscr{R}_L[\tau(t,t_0)] \equiv \overline{V(t)\,V(t_0)}/v'(t)\,v'(t_0),$$

$$V(t) \equiv \partial Y/\partial t = \text{particle (Lagrangian) velocity},$$

$$\tfrac{1}{2}Z^2(\tau) \equiv F(\tau) - \left(\frac{0\cdot 375}{C}\right)^2 \int_0^\tau \int_0^\beta F(\alpha)\,d\alpha\,d\beta,$$

$$F[\tau(t,t_0)] \equiv \tfrac{1}{2}\overline{Y^2}(t,t_0)/L_g(t)\,L_g(t_0),$$

$$\tau(t,t_0) \equiv C \ln(\Delta t/\Delta t_0),$$

and C is a time scale such that the local time scale $t_s(t) = L_g(t)/v'(t) = \Delta t/C$. This result can be verified by substituting the expression for Z^2 back into (1). For $\tau < 6$, the integral term in the expression for $\tfrac{1}{2}Z^2(\tau)$ is less than 3 % of $F(\tau)$.

From the inverse of (1), it can be shown that as $\tau \to 0$ (i.e. $t \to t_0$)

$$Z(\tau) \to \tau$$

and as $\tau \to \infty$

$$Z^2(\tau) \to 2T_\tau \tau,$$

where T_τ, the Lagrangian integral time scale, is defined by equation (2) below.

5. Interpretation of mean temperature profiles

It seems reasonable (and can be proved analytically; see, for example, Corrsin 1962; Saffman 1963) that, if there were no molecular diffusion, the mean temperature $\bar{\theta}$ measured at a point downstream from an ideal source (a Dirac delta-function source) would be equal to the probability that a particle be found at that point. Therefore the second moment of the mean temperature profile,

$$\int_{-\infty}^{\infty} y^2\bar{\theta}(y)\,dy \bigg/ \int_{-\infty}^{\infty} \bar{\theta}(y)\,dy,$$

is equal to $\overline{Y^2}$, the dispersion. In the absence of any non-speculative theory valid for the entire range of consideration,† the measured dispersion was corrected for

† Saffman's (1960) result is valid for $(\kappa/\nu)\,\omega\,(t-t_0) \ll 1$ in his notation, or $\tau \ll 0\cdot 09$.

molecular diffusion by assuming statistical independence of molecular and turbulent phenomena, as suggested by Taylor (1935). It is hoped that the effect of the interaction between turbulent and molecular diffusion is small. This hope is supported by Mickelsen (1960) and others, as well as by the results of these measurements.

6. Results

The dispersion was measured downstream of a line source (tagging wire) at two stages of decay (i.e. two streamwise positions in the flow, $U_0 t_0/M = 42$ and 98; see figure 1 for co-ordinate system and notation). Mean temperature profiles, in the direction perpendicular to the mean flow and the tagging wire, were taken for distances (from the tagging wire) up to 7·0 m for the $U_0 t_0/M = 42$ case and up to 5·0 m (to nearly the end of the test section) for the other case. The standard deviations of the dispersion at these positions were 5·60 cm and 4·45 cm, respectively.

In figure 2, a typical mean temperature profile is plotted, the different symbols indicating the order in which the data were taken. By plotting on probability paper the integral of the symmetrically faired curve, a Gaussian function was fitted to the data. The resulting Gaussian distribution, shown in figure 2, fits the data to within the accuracy of the measurement.

$2F(\tau)$ and $Z^2(\tau)$ are tabulated in table 1 while $Z(\tau)$ and $Z^2(\tau)$ are plotted for short and long times (figures 3(*a*) and (*b*)), respectively. The collapse of the data taken with the two tagging-wire positions is within the scatter for practically the entire range, consistent with the self-preservation assumption, and suggesting that the correction for molecular diffusion is reasonable. (The ratio of the molecular to turbulent diffusion for the same dimensionless time τ depends on the tagging time t_0. For example, for $\tau = 0{\cdot}25$, the estimated correction for molecular diffusion is 6·3 % of Z^2 if $U_0 t_0/M = 42$ and 7·8 % if $U_0 t_0/M = 98$; for $\tau = 5$, the corrections are 0·75 % and 0·89 % respectively.) Three of the last four points (largest τ) taken with $U_0 t_0/M = 98$ appear to deviate slightly beyond the scatter. No effect of resistance-thermometer wire length or tagging-wire diameter is detectable. The long-time asymptote appears to have been 'reached', and the short-time asymptote is exhibited in figure 3(*a*), having the predicted slope of 1·0. The apparent source position was found, by extrapolation, to be at $\tau = -0{\cdot}008 \pm 0{\cdot}001$.

From the measured dispersion, the Lagrangian correlation coefficient may be computed using (1). Since double differentiation of a set of data is a difficult operation, the Lagrangian micro- and integral scales α_τ and T_τ, were estimated first, and were used as guides in estimating the correlation function. The scales are defined by

$$\left.\begin{aligned} &\text{stationary Lagrangian (time) microscale:} \quad \alpha_\tau^{-2} \equiv -\tfrac{1}{2}[d^2\mathscr{R}_L(\tau)/d\tau^2]_{\tau=0}, \\ &\text{stationary Lagrangian (time) integral scale:} \quad T_\tau \equiv \int_0^\infty \mathscr{R}_L(\tau)\, d\tau. \end{aligned}\right\} \quad (2)$$

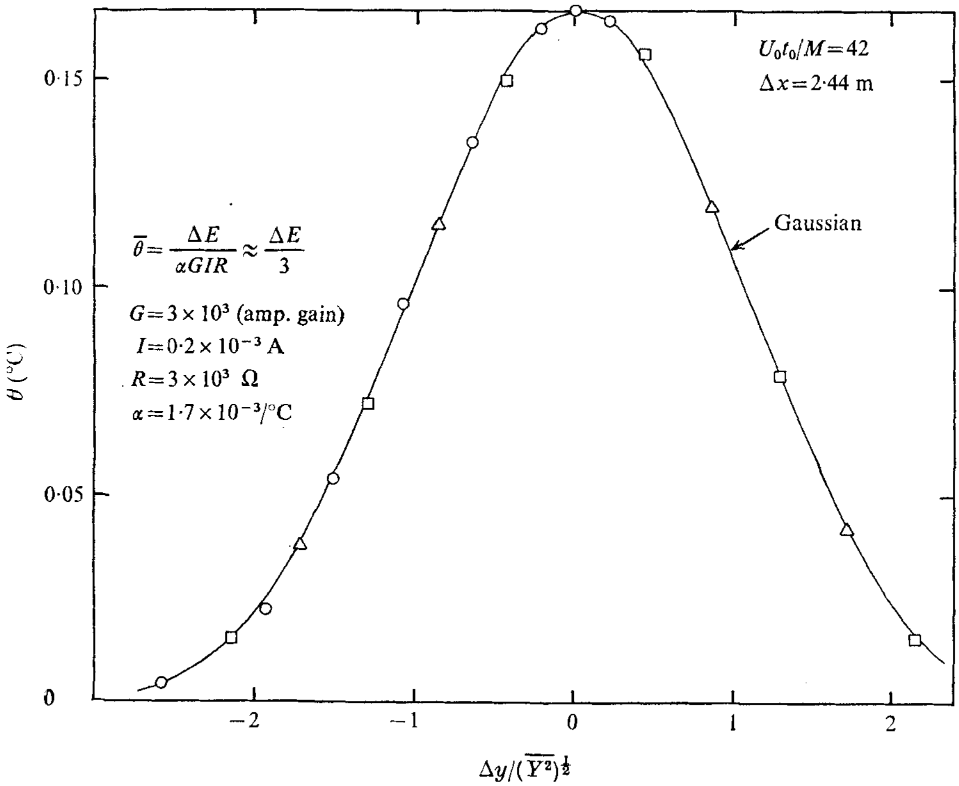

FIGURE 2. Typical mean temperature profile.

Dimensional stationary scales can be defined (Comte-Bellot & Corrsin, 1971) using a dimensional stationary time variable

$$d\mathscr{T} \equiv \frac{t_s(t_r)}{t_s(t)}\,dt = \frac{\Delta t_r}{\Delta t}\,dt, \tag{3}$$

where t_r is a reference time, to be selected. The micro- and integral time scales $\alpha_{\mathscr{T}}$, $T_{\mathscr{T}}$ corresponding to the variable time $\mathscr{T}$ will then be the time scales of the flow made stationary to correspond to the time t_r. By comparing (3) with the previously defined dimensionless time variable

$$d\tau = \frac{dt}{t_s(t)} = \frac{C}{\Delta t}\,dt$$

it is obvious that

$$\alpha_{\mathscr{T}} = (\Delta t_r/C)\,\alpha_\tau \quad \text{and} \quad T_{\mathscr{T}} = (\Delta t_r/C)\,T_\tau.$$

The actual (decaying) Lagrangian time scales will be defined subsequently.

$U_0t_0/M = 42$				$U_0t_0/M = 98$			
τ	$2F$	Z^2	Notes	τ	$2F$	Z^2	Notes
0·129	0·0167	0·0167	†	0·173	0·032	0·032	†
0·171	0·0318	0·0318	†				
0·213	0·0490	0·0490	†	0·340	0·129	0·129	†
0·254	0·0689	0·0689	†				
0·375	0·146	0·146	†	0·500	0·268	0·268	†
0·493	0·274	0·274	†‡				
0·608	0·383	0·383	†‡	0·654	0·441	0·441	†
0·608	0·386	0·386	†				
0·720	0·588	0·588	†‡	0·654	0·460	0·460	†
0·720	0·528	0·528	†				
0·829	0·694	0·694	†	0·803	0·646	0·646	†
0·935	0·857	0·856	†				
1·14	1·22	1·22	†	1·02	1·05	1·05	.
1·34	1·64	1·64	†				
1·34	1·62	1·61	†	1·22	1·39	1·38	.
1·52	2·02	2·01	†				
1·52	2·14	2·14	.	1·48	1·90	1·90	.
1·70	2·34	2·34	†				
1·70	2·57	2·57	.	1·72	2·51	2·51	.
1·70	2·50	2·50	.				
1·70	2·48	2·47	.	1·72	2·57	2·56	.
1·87	3·01	3·00	†				
1·87	2·94	2·93	†	1·95	2·98	2·97	.
1·87	3·14	3·13	.				
1·87	2·96	2·96	.	2·17	3·58	3·57	.
2·04	3·34	3·33	.				
2·04	3·35	3·34	‡	2·37	4·37	4·35	.
2·04	3·26	3·35	.				
2·20	3·88	3·87	.	2·55	4·70	4·68	.
2·35	4·22	4·20	.				
2·64	5·22	5·21	.				
2·91	5·95	5·91	.				
2·91	6·03	5·99	.				
3·17	6·77	6·71	.				
3·64	8·58	8·47	.				
4·44	11·3	11·1	.				
5·12	14·6	14·2	.				
5·70	17·0	16·5	.				

† Tagging-wire diameter = 0·0127 mm. If no dagger in note column, then diameter = 0·127 mm.

‡ Length of resistance thermometer ≈ 0·5 mm. If no double-dagger in note column, then length ≈ 5mm.

TABLE 1. Dispersion data

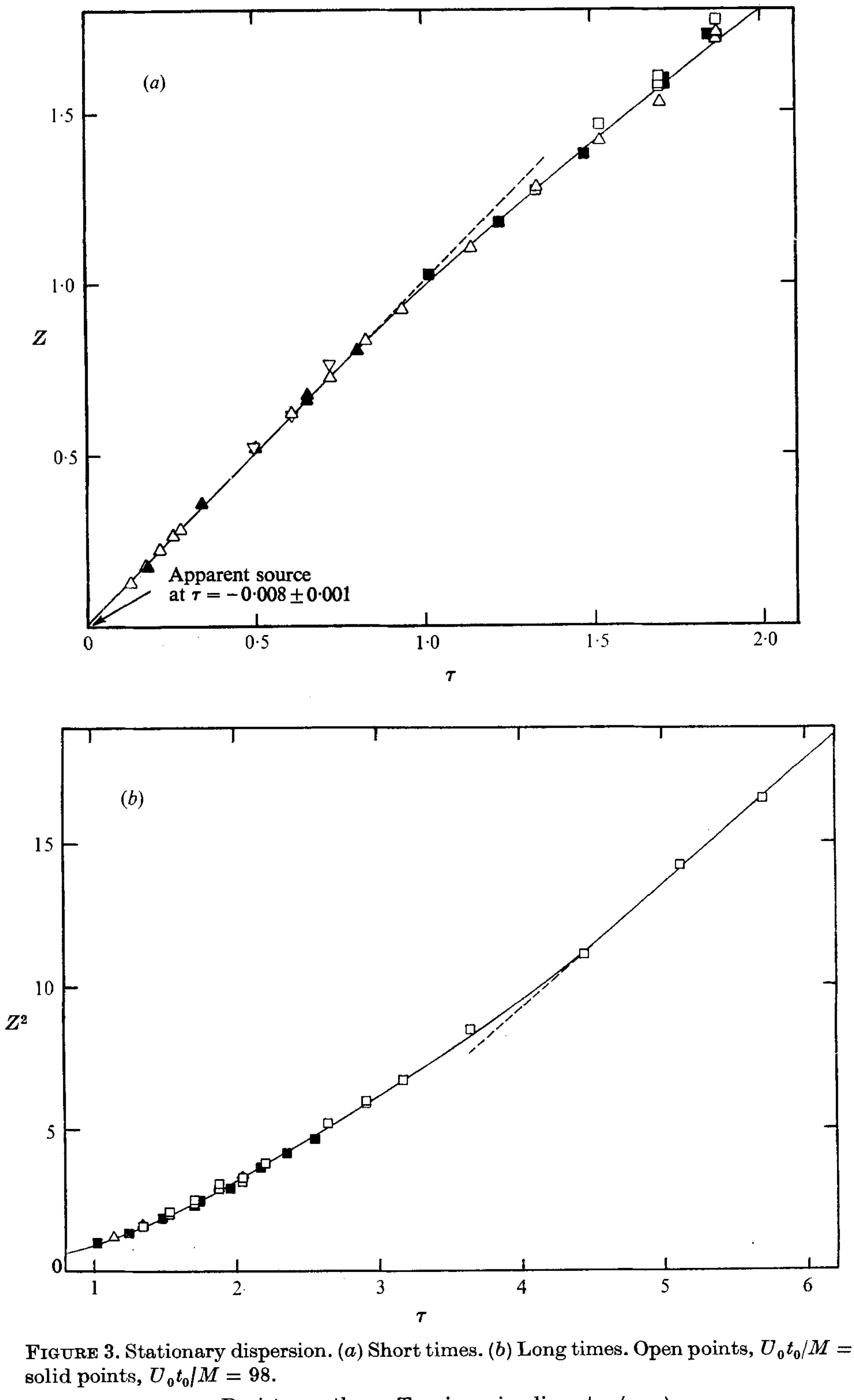

FIGURE 3. Stationary dispersion. (*a*) Short times. (*b*) Long times. Open points, $U_0t_0/M = 42$; solid points, $U_0t_0/M = 98$.

Resistance thermometer length (mm)	Tagging-wire diameter (mm) 0·127	Tagging-wire diameter (mm) 0·0127
5·0	□	△
0·5	◇	▽

Lagrangian microscale

The technique of Uberoi & Corrsin (1953) was used for computing the Lagrangian microscale from dispersion data. Since (1) can be written in the form

$$\tfrac{1}{2}Z^2(\tau) = \int_0^\tau (\tau-\tau')\mathscr{R}_L(\tau')\,d\tau',$$

then

$$\mathscr{R}_L(\tau) = 1-(\tau^2/\alpha_\tau^2)+O(\tau^4)$$

can be substituted in it:

$$\tfrac{1}{2}Z^2(\tau) \to \int_0^\tau (\tau-\tau')\left[1-\frac{\tau'^2}{\alpha_\tau^2}+O(\tau'^4)\right]d\tau'$$

$$= \frac{\tau^2}{2}\left[1-\frac{1}{6}\frac{\tau^2}{\alpha_\tau^2}+O(\tau^4)\right] \quad \text{as} \quad \tau \to 0,$$

whence

$$\alpha_\tau^2 = -\frac{1}{6}\left\{\frac{d}{d(\tau^2)}\left[\frac{Z^2(\tau)}{\tau^2}\right]+O(\tau^2)\right\}^{-1}.$$

Thus, by plotting Z^2/τ^2 against τ^2 and estimating the slope of the faired curve at $\tau^2 = 0$, the Lagrangian microscale can be computed.

In figure 4 the (stationary, dimensionless) microscale is determined by the method just described and is found to lie between 1·5 and 1·8. To calculate the microscale (of the decaying flow) corresponding to a given tagging time t_0 defined by

$$\alpha_t^{-2} \equiv -\partial^2\{\mathscr{R}_L[\tau(t,t_0)]_{t=t_0}\}/\partial t^2$$

$\partial^2/\partial t^2$ is transformed to $d^2/d\tau^2$ using the definition of τ. Since mathematics requires that $d[\mathscr{R}_L(\tau)]_{\tau=0}/d\tau = 0$ the simple relationship

$$\alpha_t = \Delta t_0\,\alpha_\tau/C = \alpha_{\mathscr{T}}$$

results. Then for $U_0t_0/M = 42$ ($t_0 = 0{\cdot}195$ s), α_t (or $\alpha_{\mathscr{T}}$) is bounded by approximately 69 and 83 ms.

Lagrangian integral scale

From its definition (2), and using (1) it is simple to show that the equivalent stationary Lagrangian integral scale is $T_\tau = \lim_{\tau\to\infty} d[\tfrac{1}{2}Z^2(\tau)]/d\tau$. Thus, by measuring the slope of the $Z^2(\tau)$ curve plotted in figure 3(*b*), the dimensionless integral time scale was found to be 2·16. Since the asymptotic slope might not have been reached, this value is considered to be a lower bound. A more precise estimate will be made in the next section.

As with the microscale, an integral time scale corresponding to a given tagging time can be calculated. If the integral time scale in the decaying flow is defined by

$$T_t(t_0) = \int_0^\infty \mathscr{R}_L\left[\tau(t_0+t',t_0)\right]dt',$$

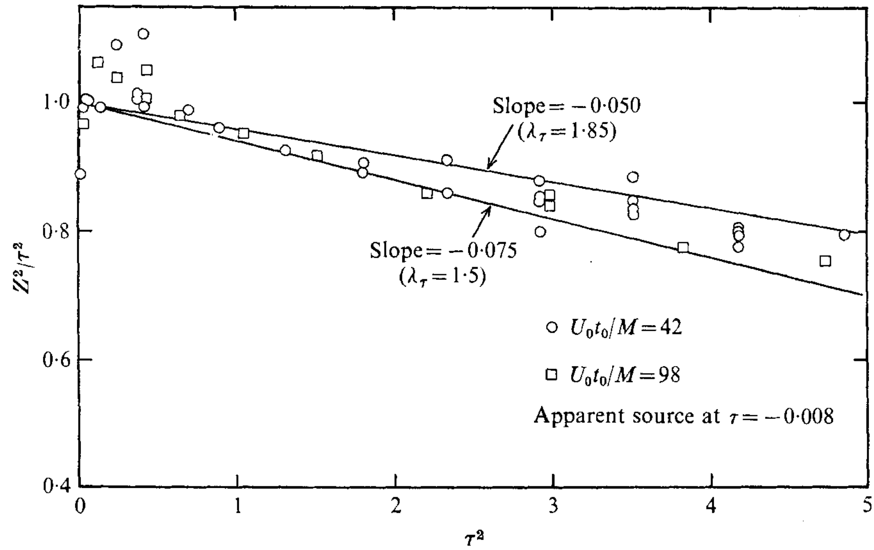

FIGURE 4. Determination of microscale.

then a change of integration variable to τ gives

$$T_t(t_0) = \frac{\Delta t}{C}\int_0^\infty \mathcal{R}_L(\tau)\, e^{\tau/C}\, d\tau. \tag{4}$$

$T_t(t_0)$ is not as simple to calculate as was α_t, since the Lagrangian correlation coefficient must first be computed. This would be avoided if (4) could be integrated by parts to give $d^2\mathcal{R}_L(\tau)/d\tau^2 = \frac{1}{2}Z^2(\tau)$. When this was attempted, the result obtained was

$$T_t(t_0) = -\Delta t_0\left\{[e^{\tau/C}\mathcal{R}_L(\tau) + C\,e^{\tau/C}(d/d\tau)\mathcal{R}_L(\tau)]_{\tau\to\infty} - C\int_0^\infty \tfrac{1}{2}Z^2(\tau)\, e^{\tau/C}\, d\tau\right\}.$$

The term involving the dispersion integral diverges (Z^2 and C are positive) and thus, supposedly, so does at least one of the other terms on the right-hand side. Therefore, the indirect method (equation (4)) must be used to obtain $T_t(t_0)$.

Lagrangian velocity correlation coefficient function

The Lagrangian velocity correlation coefficient function was computed by trial and error, using the micro- and integral scales just computed. The resulting correlation is shown in figure 5 and the subsequently computed dispersion curve is compared with the dispersion data in figure 6. The dispersion corresponding to the correlation lies almost within the scatter of the data. Considerably more effort is required to improve the fit.

To calculate the integral scale, the correlation curves were extrapolated as shown in the figure. The stationary dimensionless integral scale T_τ is 2·4, including a contribution from the extrapolated tail of 8 %. This corresponds to

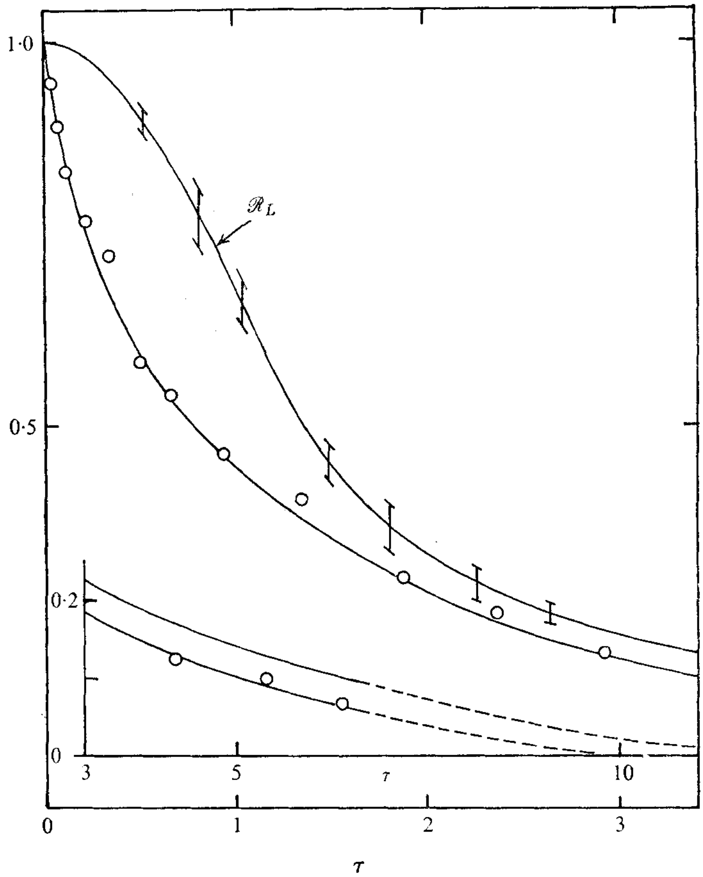

FIGURE 5. Lagrangian and Eulerian correlation coefficients. ○, Eulerian space–time correlation (Comte-Bellot & Corrsin 1971); $\mathscr{R}_L$, Lagrangian correlation (vertical lines represent range $1{\cdot}50 \leqslant \lambda_\tau \leqslant 1{\cdot}85$).

a stationary integral scale at $U_0 t_0/M = 42$ of 110 ms. The integral scale in the decaying flow was computed to be 240 ms but includes a 24 % contribution from the extrapolated tail, certainly decreasing the accuracy.

7. Comparisons of Lagrangian and Eulerian correlations and scales

Figure 5 compares the experimental Lagrangian and Eulerian correlation functions. We see that $\mathscr{R}_L(\tau) \geqslant \mathscr{R}_E(\tau)$, and that there is a strong contrast,

$$[1-\mathscr{R}_L(\tau)] \ll [1-\mathscr{R}_E(\tau)] \quad \text{for small } \tau.$$

A qualitative open question is whether there are general grounds for expecting the Lagrangian velocity correlation in time to be greater than or less than the basic Eulerian one (i.e. in an Eulerian frame in which the mean velocity is everywhere zero). The experimental results collected in figure 5 indicate that $\mathscr{R}_L(\tau) \geqslant \mathscr{R}_E(\tau)$, except possibly at large τ, where both functions $\mathscr{R}_L, \mathscr{R}_E \ll 1$.

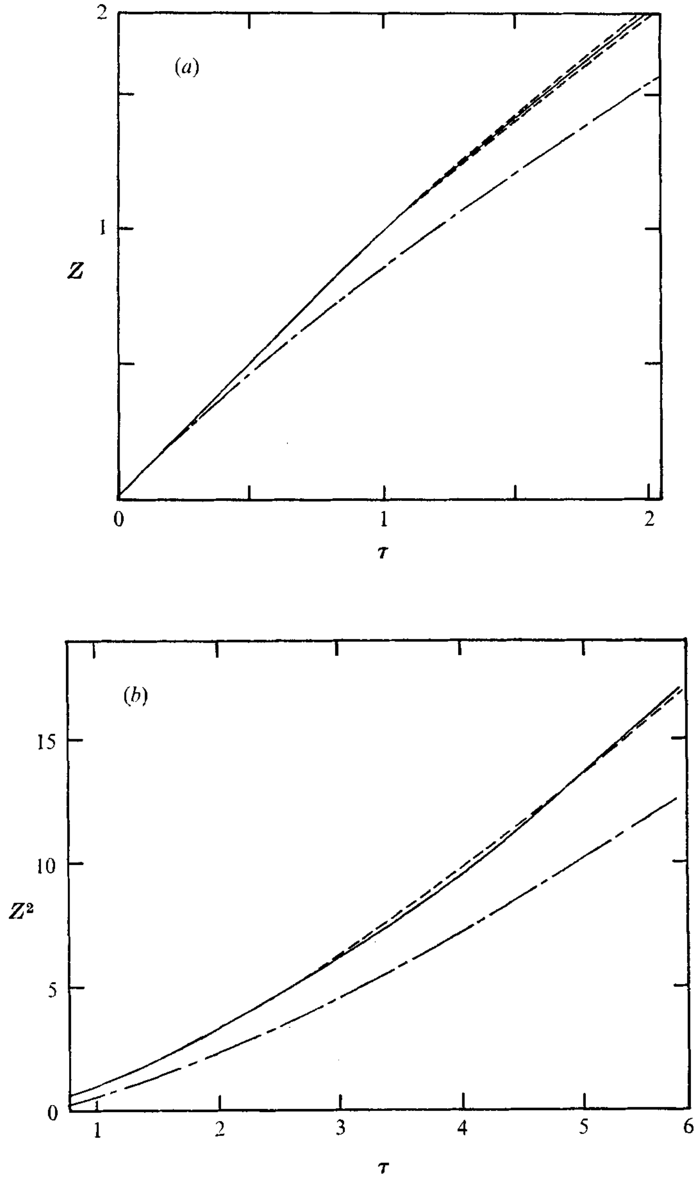

FIGURE 6. Measured and calculated dispersion. (*a*) Short times. (*b*) Long times. ——, measured; – – –, calculated from plotted Lagrangian correlation; — · —, calculated from Eulerian space–time correlation.

Although simple similarity assumptions give roughly equal time microscales, some intuitive notions suggest that the Lagrangian microscale is the larger (Corrsin 1963); this would imply $\mathscr{R}_L > \mathscr{R}_E$ for small τ. Kraichnan (1964), however, presented both intuitive arguments for the reverse behaviour, and a hypothetical random flow in which this occurred.

The most recent relevant work is the Riley & Patterson (1972) calculation of the two functions in three-dimensional, isotropic, decaying turbulence obtained by digital computation of the Navier–Stokes equations. They found $\mathscr{R}_L > \mathscr{R}_E$ for small τ and $\mathscr{R}_L < \mathscr{R}_E$ for large τ. Their time microscales were of the same order of magnitude, in contrast to the results presented here. Uncertainties in their calculation include the possible influence of the initial conditions assumed. Also, their turbulence Reynolds number R_λ is about $\frac{1}{3}$ of that used here.

Other relevant work includes the estimate based on Corrsin's 'independence hypothesis' (1959, 1962), which must be correct at large τ and may sometimes be crudely useful at smaller τ (Saffman 1963). It is easy to show that it gives $\mathscr{R}_L(\tau) \leqslant \mathscr{R}_E(\tau)$: for simplicity, we write only one space dimension, calling the Eulerian time correlation function $R_E^*(x, \tau)$ and the particle displacement probability density function $P_x(x; \tau)$. Then

$$\mathscr{R}_E(\tau) = R_E^*(0, \tau)$$

and the independence hypothesis is

$$\mathscr{R}_L(\tau) = \int_{-\infty}^{\infty} R_E^*(x, \tau)\, P_x(x; \tau)\, dx.$$

It is easy to see that, if the R_E^* isocorrelation contours are convex in x, τ space, then

$$R_E^*(x, \tau) \leqslant R_E^*(0, \tau) = R_E(\tau).$$

Substituting this inequality into the integral, and using the fact that

$$\int_{-\infty}^{\infty} P_x\, dx \equiv 1,$$

we find

$$\mathscr{R}_L(\tau) \leqslant \mathscr{R}_E(\tau).$$

Kraichnan (1970) gets the same result from his 'direct interaction hypothesis'.

The idea of a random walk with both Lagrangian and Eulerian properties was introduced by Lumley & Corrsin (1959), and studied in some detail by Patterson & Corrsin (1966). It is interesting to ask about the relative shapes of $\mathscr{R}_L(\tau)$ and $\mathscr{R}_E(\tau)$ for this simpler stochastic process. They did not seek the general mathematical answer, but ensemble-averaged machine computations on seven different ensembles showed a tendency towards

$$\mathscr{R}_L(\tau) \approx R_E^*(u'\tau, \tau) \leqslant \mathscr{R}_E(\tau)$$

for small and moderate τ, and

$$\mathscr{R}_L(\tau) \approx \mathscr{R}_E(\tau) \quad (\text{with} \quad \mathscr{R}_L < \mathscr{R}_E)$$

for large τ.

Finally, we should mention the *ad hoc* turbulent estimate by Frenkiel (1948; see also Peskin 1965), which also resulted in the Lagrangian time microscale being smaller than the Eulerian one, corresponding to $\mathscr{R}_L < \mathscr{R}_E$ for small τ.

The measured Lagrangian and Eulerian time scales (table 2) are compared with theoretical estimates in table 3. These estimates assume large turbulence Reynolds numbers R_λ, so that the 'inertial subrange' form for the various spectra, $\phi \propto \epsilon/\omega^2$ (ϵ is the dissipation rate and ω the frequency), could be used to

	Lagrangian time (ms)	Eulerian length (cm)†	Eulerian time (ms)†
Microscale	$\alpha_{\mathscr{T}} = 76$	$\lambda_g = 0{\cdot}484$	$\beta_E = 6{\cdot}2$
Integral scale	$T_{\mathscr{T}} = 110$	$L_g = 1{\cdot}27$ ($L_f = 2{\cdot}40$)	$T_E = 84$

† Measured by Comte-Bellot & Corrsin (1971).

TABLE 2. Measured scales and notation; $R_{\lambda_g} = 71{\cdot}6$, $u' = 22{\cdot}2$ cm/s

Prediction (all ratios $O(1)$)	Values calculated from data	Reference for prediction
$\dfrac{\alpha_{\mathscr{T}} u'}{\lambda_g}\left(\dfrac{30}{R_{\lambda_g}}\right)^{\frac{1}{2}}$	2·3	Uberoi & Corrsin (1953)
$\dfrac{\alpha_{\mathscr{T}}}{T_{\mathscr{T}}}\left(\dfrac{R_{\lambda_g}}{3}\right)^{\frac{1}{2}}$	3·4	Corrsin (1962)
$T_{\mathscr{T}} u'/L_f$	1·0	Corrsin (1963) Saffman (1963) Tennekes & Lumley (1972)
$T_{\mathscr{T}}/T_E$	1·3	Corrsin (1963)
$\alpha_{\mathscr{T}}/\beta_E$	12	Corrsin (1963)

TABLE 3. Comparison of theory with experiment

estimate $\int_0^\infty \omega^2\phi(\omega)\,d\omega$. Thus, the discrepancies are due, at least partially, to the relatively small $R_\lambda \approx 70$.

Tennekes & Lumley (1972, pp. 275–278) have noted that Corrsin's integral time scale estimates were partially inconsistent because they dealt with non-zero integral scales while putting the Lagrangian and Eulerian frequency spectra equal to zero at small frequencies. In the absence of explicit information, they suggest assuming the spectra constant from zero to the frequency which is roughly the inverse of the integral time scale. This improvement can change some estimates by a factor as large as 2·0.

It seems likely that estimates of the Lagrangian correlation function will require at least the two-point, space–time Eulerian correlation in a frame relative to which there is no mean flow [Lumley (1962) showed that correlations in the two kinds of co-ordinates are in fact uniquely related only at the functional, i.e. infinite order, level]. Yet several authors (cf. Pasquill 1962, p. 97; Philip 1967) have suggested that the Lagrangian correlation and the Eulerian spatial correlation are identical in shape with only a different scale of the independent variable. In the experiment described here, the ratio $v'T_{\mathscr{T}}/L_g$ of the integral scales is 1·9 while the ratio $v'\alpha_{\mathscr{T}}/\lambda_g$ of the microscales is 3·5. The former ratio is within a factor of two of previous measurements of the ratio of the integral scales: Snyder & Lumley (1971) obtained 3 while Pasquill (1962) suggests 4 (an average of eight values which range from 1·1 to 8·5). However, the disparity between the micro- and integral scale ratios indicates that the original assumption of similarity of shape is incorrect. The principal point to be made is that there is no basic reason to expect the Lagrangian correlation in time to be directly related to the Eulerian correlation in space. Philip (1967) suggested an extension dependent on the turbulence level. By dimensional reasoning, Snyder & Lumley (1971) pointed out that the correlations cannot be similar in shape.

8. Conclusions

(i) The mean temperature profile behind a heating wire spanning an isotropic turbulence (approximately the probability density of the lateral displacement of a fluid particle) is Gaussian within the accuracy of measurement, supporting earlier results. The maximum scatter on a typical profile is less than 2 % of the peak.

(ii) The measured Lagrangian velocity microscale of time is much larger than the Eulerian microscale of the turbulent velocity observed in a frame moving with the mean flow, reported by Comte-Bellot & Corrsin (1971).

(iii) The stationary Lagrangian time integral scale is about 30 % larger than that of the Eulerian correlation moving with the mean flow.

(iv) The Lagrangian velocity correlation coefficient was calculated and compared with the simplest Eulerian velocity correlation in time, i.e. that moving with the mean flow (figure 5). As implied by (ii) and (iii), they are rather different in shape.

(v) A realistic assumption of self-preservation of the particle velocities was made to calculate the Lagrangian correlation coefficient. This assumption collapsed the data taken with the source at two different stages of decay, verifying the self-preservation assumption. The recasting of the results into those for a (hypothetical) non-decaying isotropic turbulence should be useful for comparison with future theories; theories tend to be simpler for non-decaying isotropic turbulence.

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